

i を虚数単位とし, $\alpha = \frac{\sqrt{2}(-1+i)}{2}$ とする. このとき $\alpha^2 = \boxed{(\text{工})}$ であり, $\alpha^{211} = \boxed{(\text{才})}$ である.

(17 慶應大 看護医療 1(2))

〔答〕	(工)	(才)
$-i$	$\frac{\sqrt{2}(1+i)}{2}$	

【解答】

$$\alpha = \frac{\sqrt{2}(-1+i)}{2} \text{ であるから}$$

$$\alpha^2 = \frac{1}{2}(1 - 2i + i^2) = \boxed{-i} \quad \dots\dots (\text{工の答})$$

$$\alpha^4 = (-i)^2 = -1$$

よって

$$\begin{aligned} \alpha^{211} &= \alpha^{4 \cdot 52 + 3} = (\alpha^4)^{52} \cdot \alpha^3 = (-1)^{52} \cdot \alpha^2 \cdot \alpha = 1 \cdot (-i) \cdot \alpha \\ &= \frac{\sqrt{2}(i - i^2)}{2} = \boxed{\frac{\sqrt{2}(1+i)}{2}} \quad \dots\dots (\text{才の答}) \end{aligned}$$

• $\alpha = \frac{\sqrt{2}(-1+i)}{2} = \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}$ であり, ド・モアブルの定理を用いると

$$\alpha^2 = \cos \left(2 \times \frac{3\pi}{4} \right) + i \sin \left(2 \times \frac{3\pi}{4} \right) = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} = -i$$

$$\begin{aligned} \alpha^{211} &= \cos \left(211 \times \frac{3\pi}{4} \right) + i \sin \left(211 \times \frac{3\pi}{4} \right) \\ &= \cos \left(158\pi + \frac{\pi}{4} \right) + i \sin \left(158\pi + \frac{\pi}{4} \right) \\ &= \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \end{aligned}$$