次の式を計算せよ．

$$\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)^2+\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2-\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)^2-\left(\frac{\sqrt{5}-1}{\sqrt{5}+1}\right)^2$$

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【答】 $3\sqrt{5}+24\sqrt{2}$

【解答】

項の順序をかえ，分母を有理化してから式を整理すると

(与式)

$$=\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)^2-\left(\frac{\sqrt{5}-1}{\sqrt{5}+1}\right)^2+\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2-\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)^2$$

$$=\left(\frac{6+2\sqrt{5}}{5-1}\right)^2-\left(\frac{6-2\sqrt{5}}{5-1}\right)^2+\left(\frac{3+2\sqrt{2}}{2-1}\right)^2-\left(\frac{3-2\sqrt{2}}{2-1}\right)^2 \quad \text{(分母の有理化)}$$

$$=\left(\frac{3+\sqrt{5}}{2}\right)^2-\left(\frac{3-\sqrt{5}}{2}\right)^2+(3+2\sqrt{2})^2-(3-2\sqrt{2})^2$$

$$=\frac{6\cdot 2\sqrt{5}}{4}+6\cdot 4\sqrt{2} \quad (\because\ A^2-B^2=(A+B)(A-B))$$

$$=\boldsymbol{3\sqrt{5}+24\sqrt{2}} \qquad \cdots\cdots(\text{答})$$

となる．