

次の式を簡単にせよ.

$$(1) (\cos^2 \theta + \sin^2 \theta)^3 - (\cos^6 \theta + \sin^6 \theta)$$

$$(2) \frac{2(\cos^6 \theta + \sin^6 \theta) + 1}{3(\cos^4 \theta + \sin^4 \theta)}$$

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【答】

$$(1) 3 \cos^2 \theta \sin^2 \theta$$

$$(2) 1$$

【解答】

(1) 第 1 項を展開すると

$$\begin{aligned} & (\cos^2 \theta + \sin^2 \theta)^3 - (\cos^6 \theta + \sin^6 \theta) \\ &= (\cos^6 \theta + 3 \cos^4 \theta \sin^2 \theta + 3 \cos^2 \theta \sin^4 \theta + \sin^6 \theta) - (\cos^6 \theta + \sin^6 \theta) \\ &= 3 \cos^2 \theta \sin^2 \theta (\cos^2 \theta + \sin^2 \theta) \\ &= \mathbf{3 \cos^2 \theta \sin^2 \theta} \qquad \dots\dots(\text{答}) \end{aligned}$$

となる.

(2) (1) より

$$\begin{aligned} & \frac{2(\cos^6 \theta + \sin^6 \theta) + 1}{3(\cos^4 \theta + \sin^4 \theta)} \\ &= \frac{2\{(\cos^2 \theta + \sin^2 \theta)^3 - 3 \cos^2 \theta \sin^2 \theta\} + 1}{3\{(\cos^2 \theta + \sin^2 \theta)^2 - 2 \cos^2 \theta \sin^2 \theta\}} \\ &= \frac{2(1 - 3 \cos^2 \theta \sin^2 \theta) + 1}{3(1 - 2 \cos^2 \theta \sin^2 \theta)} \\ &= \frac{3 - 6 \cos^2 \theta \sin^2 \theta}{3(1 - 2 \cos^2 \theta \sin^2 \theta)} \\ &= \mathbf{1} \qquad \dots\dots(\text{答}) \end{aligned}$$

である.