

極限 $\lim_{n \rightarrow \infty} \frac{\sqrt{n+2} - \sqrt{n+3}}{\sqrt{3n+2} - \sqrt{3n+3}}$ の値は である.

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【答】

$\sqrt{3}$

【解答】

与式を変形すると

$$\begin{aligned}
 & \lim_{n \rightarrow \infty} \frac{\sqrt{n+2} - \sqrt{n+3}}{\sqrt{3n+2} - \sqrt{3n+3}} \\
 = & \lim_{n \rightarrow \infty} \left\{ \frac{(\sqrt{n+2} - \sqrt{n+3})(\sqrt{n+2} + \sqrt{n+3})}{\sqrt{n+2} + \sqrt{n+3}} \right. \\
 & \quad \left. \times \frac{\sqrt{3n+2} + \sqrt{3n+3}}{(\sqrt{3n+2} - \sqrt{3n+3})(\sqrt{3n+2} + \sqrt{3n+3})} \right\} \\
 = & \lim_{n \rightarrow \infty} \left\{ \frac{(n+2) - (n+3)}{\sqrt{n+2} + \sqrt{n+3}} \cdot \frac{\sqrt{3n+2} + \sqrt{3n+3}}{(3n+2) - (3n+3)} \right\} \\
 = & \lim_{n \rightarrow \infty} \frac{\sqrt{3n+2} + \sqrt{3n+3}}{\sqrt{n+2} + \sqrt{n+3}} \\
 = & \lim_{n \rightarrow \infty} \frac{\sqrt{3 + \frac{2}{n}} + \sqrt{3 + \frac{3}{n}}}{\sqrt{1 + \frac{2}{n}} + \sqrt{1 + \frac{3}{n}}} \\
 = & \frac{\sqrt{3} + \sqrt{3}}{1 + 1} \\
 = & \sqrt{3}
 \end{aligned}$$

……(答)

である.