

積分

$$\int_0^{\frac{\pi}{4}} (x^2 + 1) \cos 2x \, dx$$

を求めよ.

(21 學習院大 理・文 3(1))

【答】 $\frac{\pi^2}{32} + \frac{1}{4}$

【解答】

部分積分法を用いる.

$$\begin{aligned}
 & \int_0^{\frac{\pi}{4}} (x^2 + 1) \cos 2x \, dx \\
 &= \left[(x^2 + 1) \cdot \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 2x \cdot \frac{\sin 2x}{2} \, dx \\
 &= \frac{1}{2} \left(\frac{\pi^2}{16} + 1 \right) - \int_0^{\frac{\pi}{4}} x \sin 2x \, dx \\
 &= \frac{1}{2} \left(\frac{\pi^2}{16} + 1 \right) - \left[x \left(-\frac{\cos 2x}{2} \right) \right]_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} 1 \cdot \left(-\frac{\cos 2x}{2} \right) \, dx \\
 &= \frac{1}{2} \left(\frac{\pi^2}{16} + 1 \right) - 0 - \left[\frac{\sin 2x}{4} \right]_0^{\frac{\pi}{4}} \\
 &= \frac{1}{2} \left(\frac{\pi^2}{16} + 1 \right) - \frac{1}{4} \\
 &= \frac{\pi^2}{32} + \frac{1}{4}
 \end{aligned}
 \quad \cdots\cdots(\text{答})$$

である.