

次の定積分を求めよ.

$$I = \int_0^1 x^2 \sqrt{1-x^2} dx$$

(21 神戸大 理 2(1))

【答】 $\frac{\pi}{16}$

【解答】

$x = \sin t$ とおくと

$$dx = \cos t dt \quad \begin{array}{l|l} x & 0 \longrightarrow 1 \\ t & 0 \longrightarrow \frac{\pi}{2} \end{array}$$

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \sin^2 t \sqrt{1-\sin^2 t} \cdot \cos t dt \\ &= \int_0^{\frac{\pi}{2}} \sin^2 t \cos^2 t dt \quad (\because 0 \leq t \leq \frac{\pi}{2} \text{ で } \cos t \geq 0) \\ &= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} \sin 2t\right)^2 dt \\ &= \frac{1}{4} \int_0^{\frac{\pi}{2}} \frac{1-\cos 4t}{2} dt \\ &= \frac{1}{8} \left[t - \frac{\sin 4t}{4} \right]_0^{\frac{\pi}{2}} \\ &= \frac{\pi}{16} \end{aligned} \quad \dots\dots(\text{答})$$

である.