方程式 $2^x - (\sqrt{2})^{x+1} - 4 = 0$ を解け.

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【答】 $x = 3$

【解答】

$$2^x - (\sqrt{2})^{x+1} - 4 = 0$$

$2^x = \{(\sqrt{2})^2\}^x = \{(\sqrt{2})^x\}^2$ なので，与えられた方程式は

$$\{(\sqrt{2})^x\}^2 - \sqrt{2}(\sqrt{2})^x - 4 = 0$$

$$\{(\sqrt{2})^x - 2\sqrt{2}\}\{(\sqrt{2})^x + \sqrt{2}\} = 0$$

であり，$(\sqrt{2})^x > 0$ なので

$$(\sqrt{2})^x = 2\sqrt{2} \qquad \therefore \quad (\sqrt{2})^x = (\sqrt{2})^3$$

$$\therefore \quad \boldsymbol{x = 3}$$ ……(答)

である.