

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \sin^2 3x \, dx = \boxed{(\text{こ})}$$

(22 茨城大 後工 1(6)(ii))

【答】

(こ)
$\frac{\pi}{36}$

【解答】

半角の公式より

$$\begin{aligned} \int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \sin^2 3x \, dx &= \frac{1}{2} \int_{\frac{\pi}{18}}^{\frac{\pi}{9}} (1 - \cos 6x) \, dx \\ &= \frac{1}{2} \left[x - \frac{1}{6} \sin 6x \right]_{\frac{\pi}{18}}^{\frac{\pi}{9}} \\ &= \frac{1}{2} \left\{ \left(\frac{\pi}{9} - \frac{1}{6} \sin \frac{2\pi}{3} \right) - \left(\frac{\pi}{18} - \frac{1}{6} \sin \frac{\pi}{3} \right) \right\} \\ &= \frac{1}{2} \cdot \frac{\pi}{18} \\ &= \frac{\pi}{36} \end{aligned} \quad \dots\dots(\text{答})$$

である.