次の定積分を求めよ．

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{x}{\cos^2 x}\,dx$$

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【答】 $\dfrac{4\sqrt{3}-3}{12}\pi - \dfrac{1}{2}\log 2$

【解答】

部分積分法を用いる．

$$\begin{aligned}
\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{x}{\cos^2 x}\,dx &= \Big[x\tan x\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{3}} - \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} 1\cdot\tan x\,dx \\
&= \left(\frac{\pi}{3}\cdot\sqrt{3} - \frac{\pi}{4}\cdot 1\right) + \Big[\log|\cos x|\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \\
&= \frac{4\sqrt{3}-3}{12}\pi + \left(\log\frac{1}{2} - \log\frac{1}{\sqrt{2}}\right) \\
&= \frac{4\sqrt{3}-3}{12}\pi + \left(-\log 2 + \frac{1}{2}\log 2\right) \\
&= \boldsymbol{\frac{4\sqrt{3}-3}{12}\pi - \frac{1}{2}\log 2} \qquad \cdots\cdots(\text{答})
\end{aligned}$$

である．